Back to (not quite so) Basics

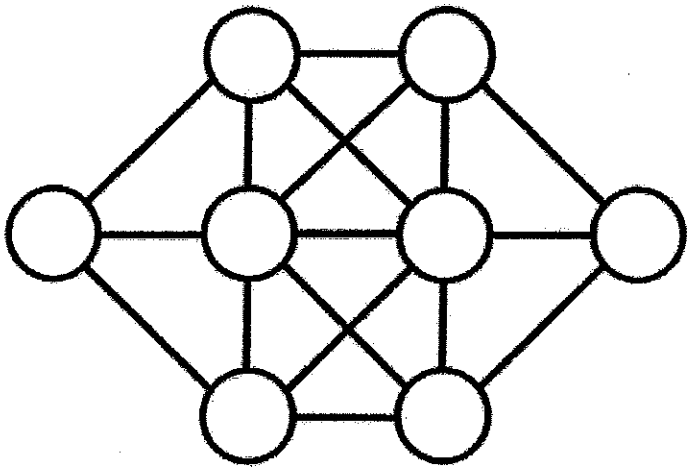
As you work through these, feel free to go off on tangents!

1. **Warm Up.**

   (a) Lianna makes two 4-digit numbers using each of the digits $1, 2, 3, 4, 5, 6, 7$, and 8 exactly once. If Lianna makes the numbers so that adding them gives the smallest possible total, what is that total?

   (b) How many possible pairs of numbers could give you the total you came up with in (a)?

2. (a) Place each of the numbers $1, 2, 3, 4, 5, 6, 7$, and 8 in a circle in the diagram below so that no two numbers that are 1 apart are in connected circles. (For example, 2 and 3 cannot be in circles that are connected by a line.)



   (b) Are there any edges that you can *add* to the picture above to still guarantee a solution? What are the maximum number of edges that we can add to still guarantee a solution?

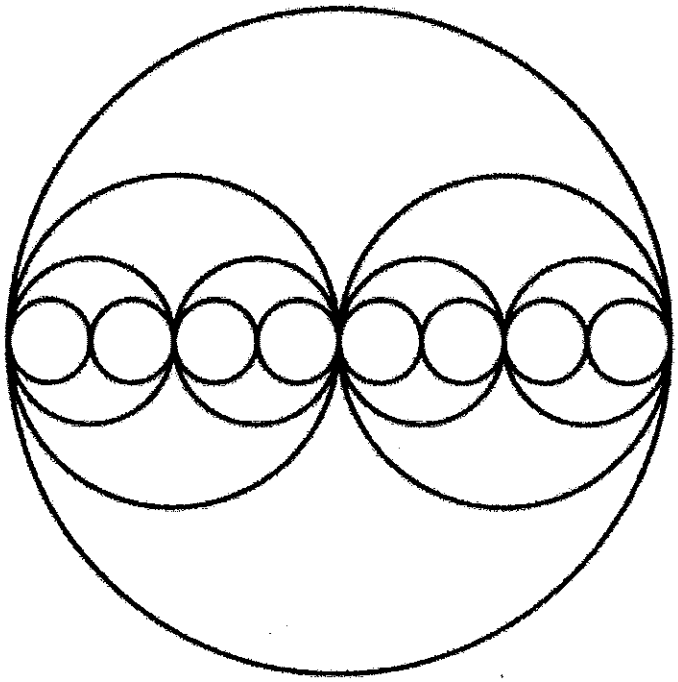
3. Let $A = 1+3+5+\ldots+99$, and $B = 2+4+6+\ldots+98$. Which is bigger, A or B, and by how much?

4. (a) Is there a 10-digit number where the first digit is equal to how many 0's are in the number, the second digit is equal to how many 1's are in the number, the third digit is equal to how many 2's are in the number, all the way up to the last digit (which is necessarily equal to the number of 9's in the number)?

# of 0s	# of 1s	# of 2s	# of 3s	# of 4s	# of 5s	# of 6s	# of 7s	# of 8s	# of 9s

(b) Is there a 10-digit number where the first digit is equal to how many 1's are in the number, the second digit is equal to how many 2's are in the number, the third digit is equal to how many 3's are in the number, all the way up to the last digit (which is necessarily equal to the number of 0's in the number)?

5. How many different ways are there to choose three circles of different sizes from this diagram so that the smallest circle you chose is inside the medium circle you chose, and the medium circle you chose is inside the largest circle you chose?



6. Take four copies of the number 1234. Rearrange the digits in each number so that the four numbers sum to 9000. [1]

[1] Problems 1-5 are from this New York Times artice https://www.nytimes.com/interactive/2017/02/17/nyregion/math-camp-quiz.html?_r=0. Some of them have been modified. Problem 6 is from this blog article by Junaid Mubeen https://brightreads.com/my-nephew-brought-home-this-menacing-maths-problem. For a generalization of Problem 4, see this paper by A. Tripathi: *A Box Filling Problem*, The Fibonacci Quarterly 27(5) (1989), 465-466.